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2)	To demostrate a function which is ~~(i)~~ one-one, onto (ii) not one-one but onto (iii) One-one but not onto		17/06/22	25/07/22	
3)	To draw the graph of $\sin^{-1}x$, using the graph of $\sin x$ and demostrate the concept of mirror reflection.		08/07/22	25/07/22	
4)	To understand the concepts of decreasing and increasing function		08/07/22	25/07/22	
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8)	Give the solution to the following activities of cartesian and vector form of presentation the equation of lines.		09/11/22	16/11/22	
9)	A furniture dealer deals in only two items namely, tables and chairs. He has ₹5000 to invest and a space to store, at the most 60 pieces. A table costs him ₹250 and a chair ₹50. He call sell a table at a profit of ₹50 and a chair at a profit of ₹15. Assuming that he can sell all the items that he buys, how should he invest his money in order that he may maximise the profit.		16/11/22	24/11/22	
10)	To explain the computation of conditions probability of a given event A, when event B has already occured, through a example of throwing a pair of dice.		28/11/22	5/12/22	

Date

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ACTIVITY - 1

Objective : To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalance relation

Materials Required : A plane paper, glue, pens, colourful threads.

Method of Construction : Take a piece of white paper of convenient size. Fix the threads randomly on the paper with the help of glue such that some of them are parallel, some are perpendicular to each other and some are inclined.

Demonstration :

- Let the threads represent the lines $l_1, l_2, \dots, l_8$
- l_1 is perpendicular to each of the lines l_2, l_3, l_4
- l_6 is perpendicular to l_7
- l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8
- (l_2, l_3) ; (l_3, l_4) ; $(l_5, l_8) \in R$

Observation :

1) In the figure, every line is parallel to itself. So the relation $R = \{(l, m) : l \parallel m\}$ is reflexive relation.

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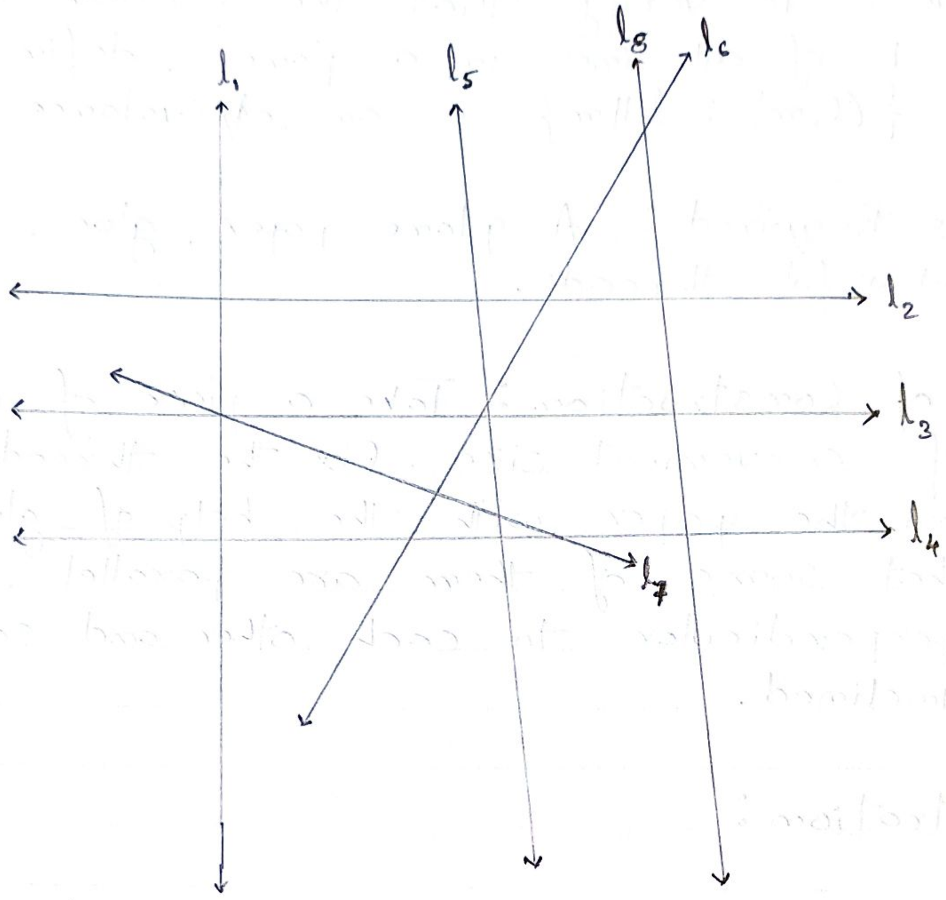
l_1
l_2
l_3
l_4
l_5
l_6
l_7
l_8

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2) In the figure, we observe that $l_2 \parallel l_3$. Also, $l_3 \parallel l_2$
So, $(l_2, l_3) \in R \Rightarrow (l_3, l_2) \in R$

Similarly, $l_3 \parallel l_4$ Also $l_4 \parallel l_3$
So, $(l_3, l_4) \in R \Rightarrow (l_4, l_3) \in R$
and $(l_5, l_8) \in R \Rightarrow (l_8, l_5) \in R$
$\therefore$ The relation R is symmetric relation.

3) In the figure, we observe that,
$l_2 \parallel l_3$ and $l_3 \parallel l_4$. Also $l_2 \parallel l_4$
So, $(l_2, l_3) \in R$ and $(l_3, l_4) \in R$
$\Rightarrow (l_2, l_4) \in R$

Similarly,

$l_3 \parallel l_4$ and $l_4 \parallel l_2$. Also $l_3 \parallel l_2$
So $(l_3, l_4) \in R$ and $(l_4, l_2) \in R$
$\Rightarrow (l_3, l_2) \in R$

Thus, the relation is transitive relation.

Hence, the relation R is reflexive, symmetric, transitive. So, R is an equivalence relation.

Application :

This activity is useful in understanding the concept of an equivalence relation.

Teacher's Signature 23.07.22

ACTIVITY - 2

Objective : To demonstrate a function which is -
(i) one - one and onto
(ii) not one - one but onto
(iii) one - one but not onto

Pre - Requisite Knowledge :

- A function $f : A \rightarrow B$ is called a one - one function if the images of distinct element of A are also distinct elements of B.

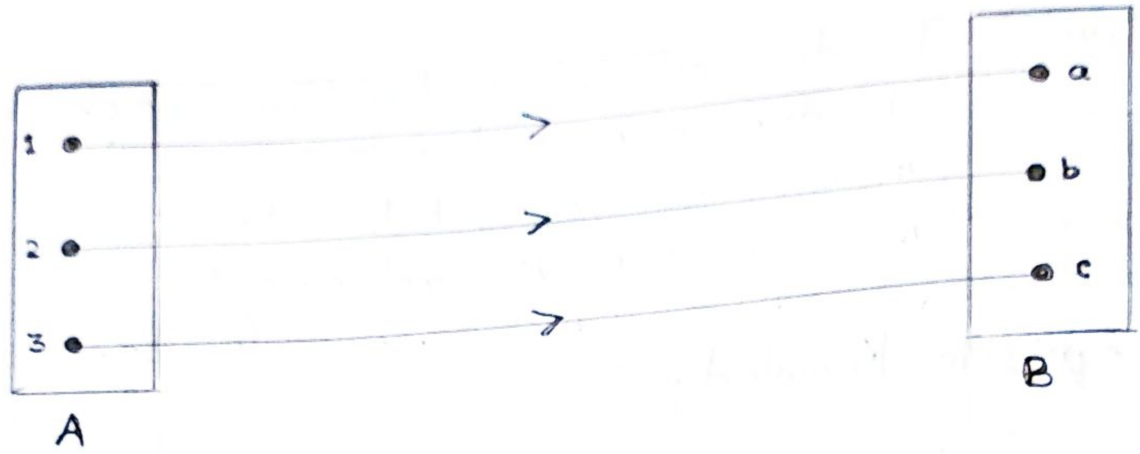
- A function $f : A \rightarrow B$ is called an onto function if every element of B is the image of at least one element of A.

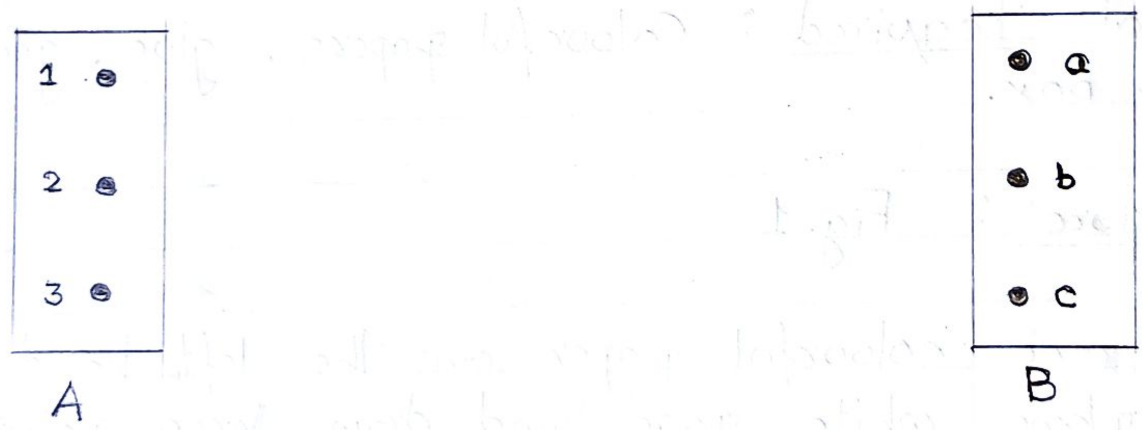
Materials Required : Colourful papers, glue, geometry box.

1. **Procedure :** Fig. 1

(a) Paste one colourful paper on the left hand side of a plane white page and draw three points on it. Name the points as 1, 2, 3.

(b) Paste another colourful paper on the right hand side of a plane white page and draw three points on it. Name the points as a, b, c.


1
2
3
A
a
b
c
B


1
2
3
A
a
b
c
B

(c) Join the points on the left side to right side.

(d) Take $A = \{1, 2, 3\}$; $B = \{a, b, c\}$

(e) Join correspondence elements of A to that of B.

Result :

The function is one - one and onto.

2. Procedure : Fig. 2

(a) Paste one colourful paper on the left hand side of a plane white sheet and draw three points on it. Name these points as 1, 2, 3

(b) Paste another colourful paper on the right hand side of the sheet and draw two points in it. Name these points as a, b.

(c) Join the points on left side to right side.

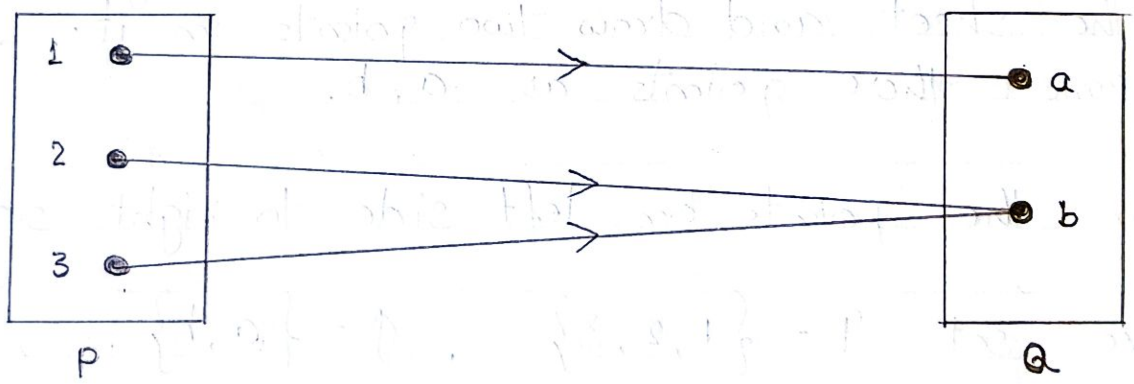
(d) Take set $P = \{1, 2, 3\}$, $Q = \{a, b\}$

(e) Join Correspondence elements of P to that of Q.

Observation :

(a) The image of the element 1 of P in Q is a


1
2
3
P
a
b
Q


1
2
3
P
a
b
Q

(b) The image of the element 2 of P in Q is b

(c) The image of the element 3 of P in Q is b.
So, figure 2 represents a function.

(d) Every element in P has a non-distinct image in Q
So, the function is not one-one.

(e) The pre-image of each element of Q in P exists.
So, the function is onto.

Result :

The function is not one-one but onto.

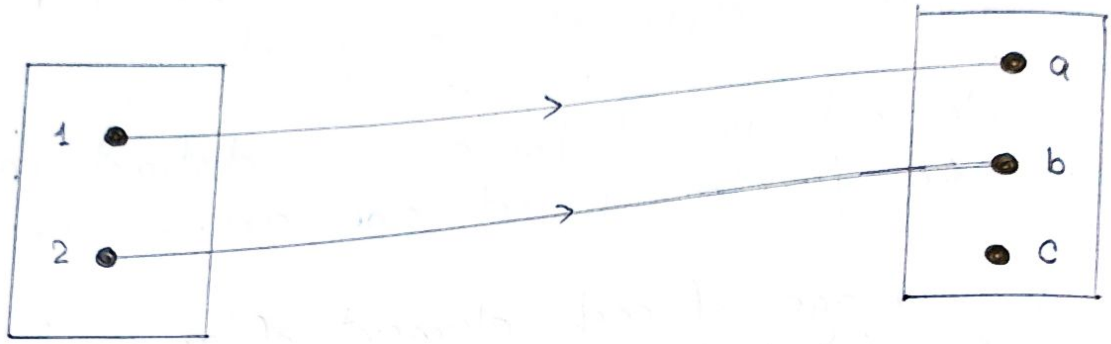
3. **Procedure :** Fig. 3

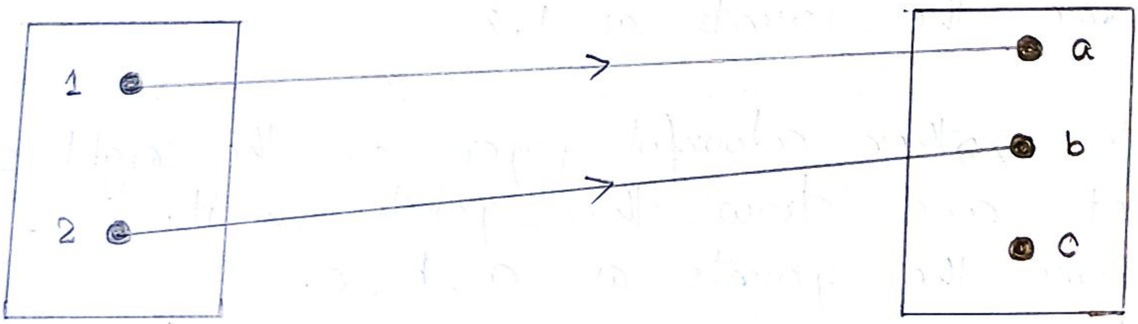
(a) Paste one colourful paper on the left side of a plane white sheet and draw two points on it.
Name the points as 1, 2

(b) Paste another colourful paper on the right side of the sheet and draw three points on it.
Name the points as a, b, c.

(c) Join the points on left side and right side

(d) Take set $X = \{1, 2\}$; $Y = \{a, b, c\}$


1
2
a
b
c


1
2
a
b
c

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(e) Join correspondence elements of X to that of Y.

Observation :

(a) The image of element 1 of X in Y is a

(b) The image of element 2 of X in Y is b.
So, figure 3 is a function.
But for the element c in Y, there is no pre-image in X

(c) Every element in X has a distinct image in Y. So, the function is one - one.

(d) The pre-image of each element of Y in X doesn't exist so the function is not onto.

Result :

The function is one - one but not onto

Application :

The activity can be used to demonstrate the concept of one - one and onto function.

25.08.22

Teacher's Signature

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ACTIVITY - 3

Objective: To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$)

Pre-Requisite Knowledge: Knowledge of trigonometric functions and their properties.

Materials Required: A hardboard of dimensions 30 cm × 30 cm ruler, coloured pencils, board pins, paper pins and strings.

Procedure:

i) Place a chart paper firmly on a hardboard with the help of board pins as shown in fig. 3.1

ii) On the chart paper, draw perpendicular axis $x'ox$ and YOY' as shown in fig. 3.2.

iii) Mark the points on y-axis $1, \frac{1}{2}, 0, -\frac{1}{2}, -1$, etc. as shown in fig 3.3.

iv) To sketch the graph of $y = \sin x$, we can say a table of values that we can compute exactly.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1

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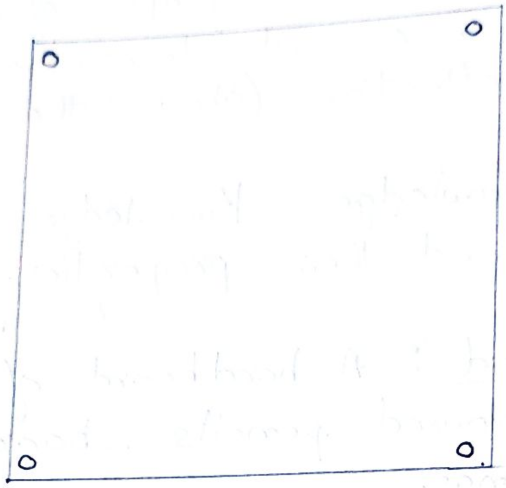
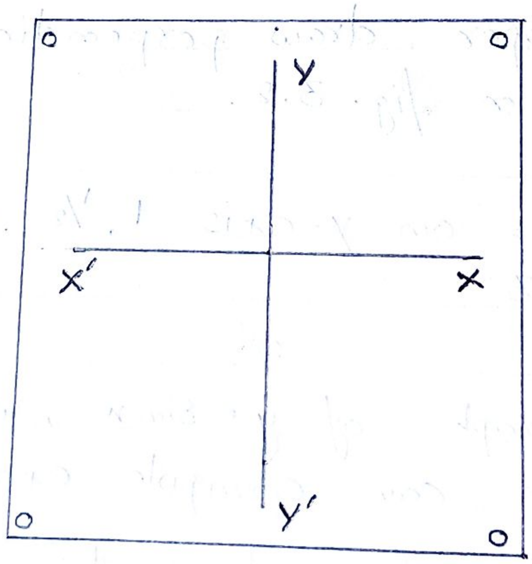



Y
X'
X
Y'

v) Fix paper pins in the coordinate plane to represent the points namely $A_1 (\pi/6, 1/2)$, $A_2 (\pi/4, 0.71)$, $A_3 (\pi/3, 0.87)$, $A_4 (\pi/4, 1)$ as shown in fig 3.3

vi) On the other side of the x-axis, repeat the same process and mark the points given in the table below:

x	$-\pi/6$	$-\pi/4$	$-\pi/3$	$-\pi/2$
$\sin x$	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1

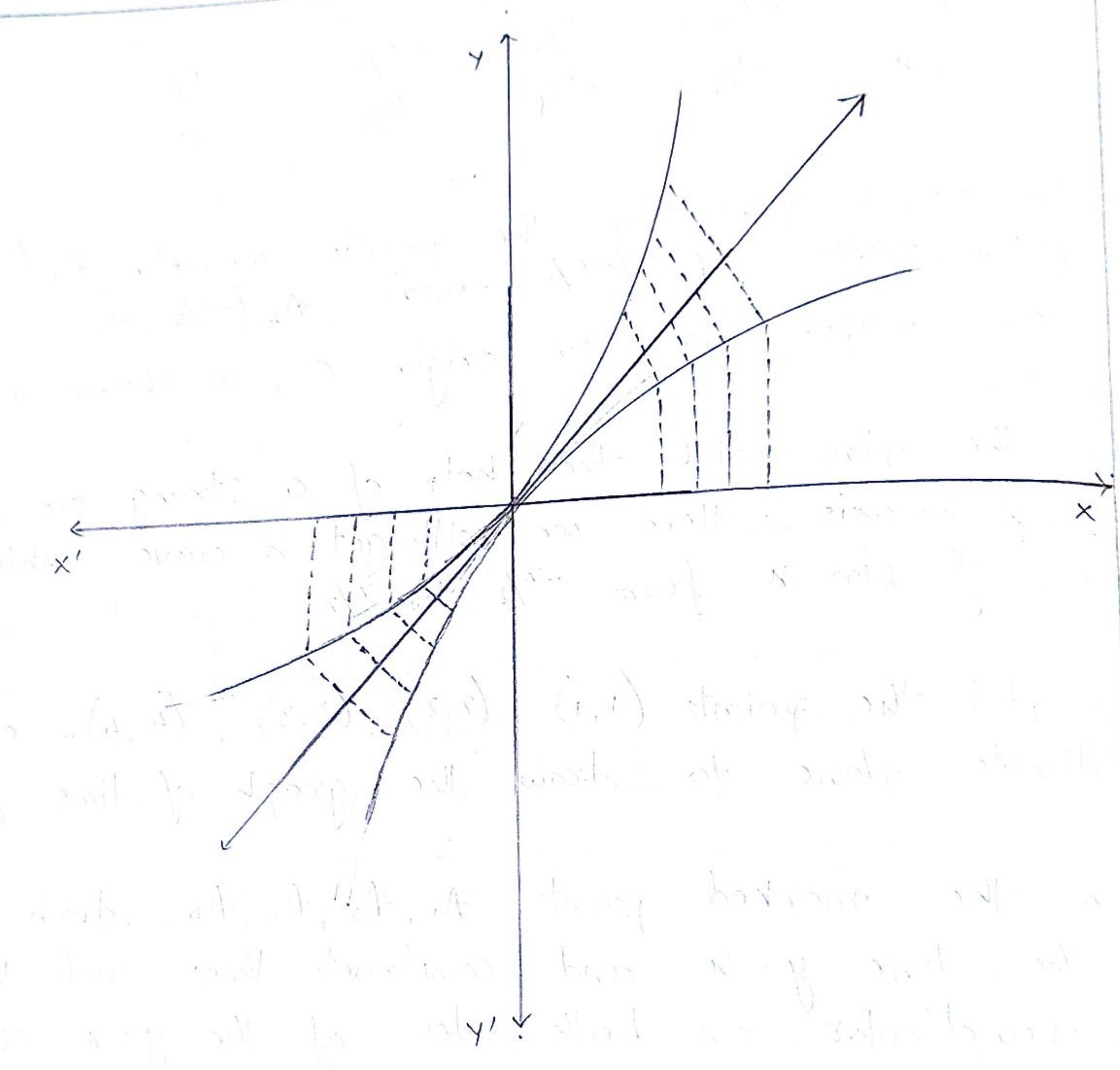
vii) Fix the paper pins on the points namely $A_1 (-\pi/6, -0.5)$, $A_2 (-\pi/4, -0.71)$, $A_3 (-\pi/3, -0.87)$, $A_4 (-\pi/2, -1)$
Fix one paper pin on origin O, as shown in fig 3.3.

viii) Join the pins with the help of a string on both side of x-axis. Here we will get a curve which is the graph of $\sin x$ from $-\pi/2$ to $\pi/2$

ix) Now plot the points (1,1), (2,2), (3,3), (4,4)... etc on the coordinate plane to draw the graph of line $y = x$.

x) From the marked points A_1, A_2, A_3, A_4, draw perpendicular on the line $y = x$ and construct lines such that length of perpendicular on both sides of the $y = x$ are equal. Mark these points as B_1, B_2, B_3, B_4 and fix the pins on them.

xi) Repeat the same process on the other side of x-axis and fix the pins on the points namely B_1', B_2', B_3', B_4'


Y
X
X'
Y'

xii) Join the pins on the sides of the line $y = x$ by a string tightly to obtain the graph of $y = \sin^{-1}x$.

xiii) Now place a mirror on the line $y = x$. The mirror image of the graph of $\sin x$ represents the graph of $\sin^{-1}x$, which shows $\sin^{-1}x$ is a reflection of $\sin x$ about the line $y = x$.

Observations :

i) The image of point A_1 in the mirror (the line $y=x$) is B_1

ii) The image of point A_2 in the mirror (the line $y=x$) is B_2

iii) The image of point A_3 in the mirror (the line $y=x$) is B_3

iv) The image of point A_4 in the mirror (the line $y=x$) is B_4

v) The image of point A_1' in the mirror (the line $y=x$) is B_1'

vi) The image of point A_2' in the mirror (the line $y=x$) is B_2'

vii) The image of point A_3' in the mirror (the line $y=x$) is B_3'

viii) The image of point A_4' in the mirror (the line $y=x$) is B_4'

Conclusion: The mirror image of the graph of $\sin x$ about the line $y = x$ is the graph of $\sin^{-1} x$, and the mirror image of the graph of $\sin^{-1} x$ about the line $y = x$ is the graph of $\sin x$

Application: This activity is useful for concept clarity about graphs of inverse trigonometric function.

25.07.22

Activity - 4

Objective : To understand the concept of decreasing and increasing function.

Pre - Requisite Knowledge :

Define - Increasing and Decreasing function.

Materials Required : A white sheet, geometry box, Glue, trigonometric table.

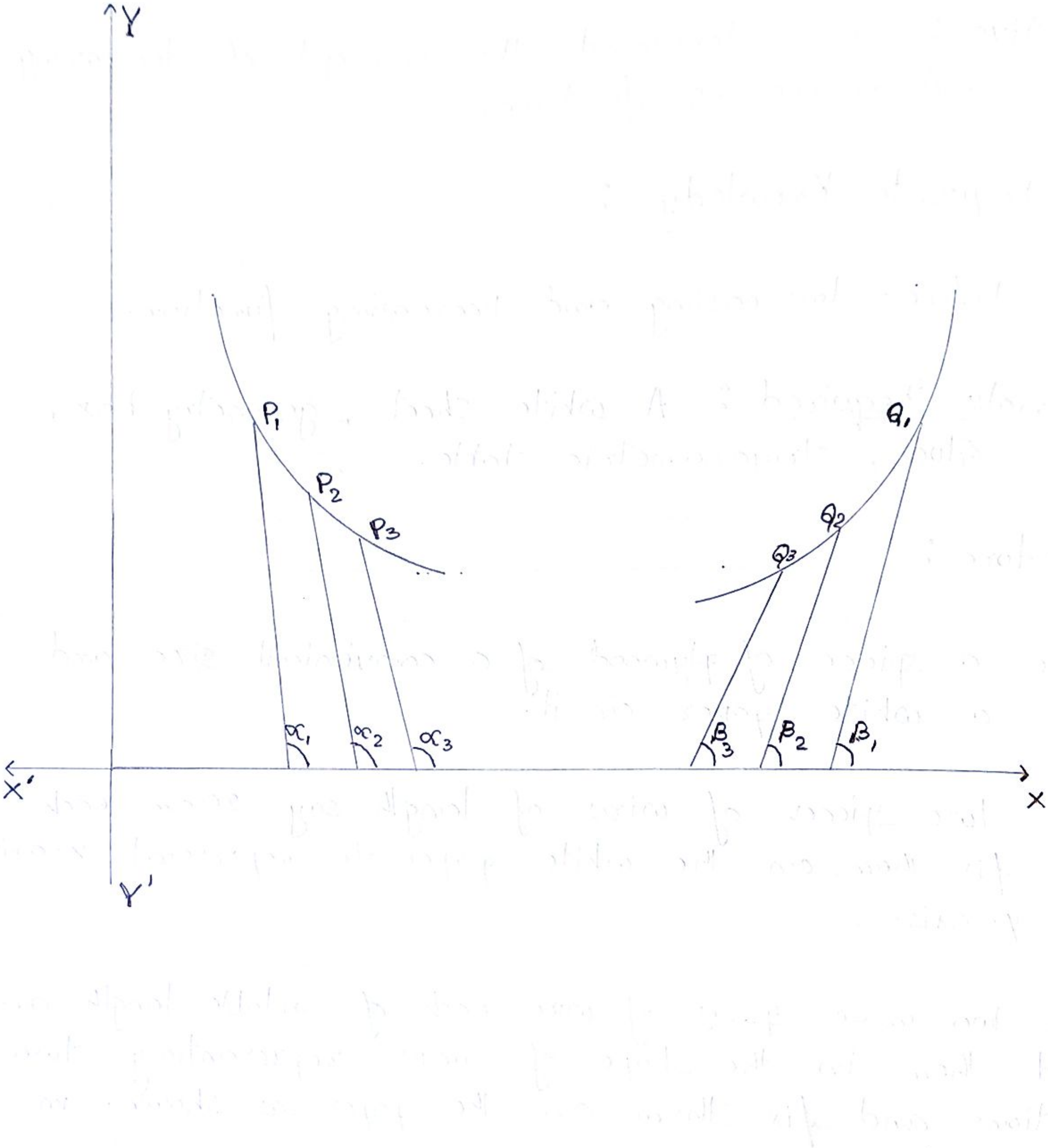
Procedure :

i) Take a piece of plywood of a convenient size and paste a white paper on it.

ii) Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent x-axis and y-axis.

iii) Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the fig 4.1.

iv) Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.



Y
X'
X
Y'
P1
P2
P3
Q1
Q2
Q3
α1
α2
α3
β3
β2
β1

Demonstration :

i) Take one straight wire and place it on the curve (on the left) such that it is tangent to the curve at the point say P_1 and making an angle α_1 with the positive direction of x-axis.

ii) α_1 is an obtuse angle, so tan α_1 is negative, i.e, the slope of the tangent at P_1 (derivatives of the function at P_1) is negative.

iii) Take another two points say P_2 and P_3 on the same curve, and make tangents, using the same wire, at P_2 and P_3 making angles α_2 and α_3, respectively with the positive direction of x-axis.

iv) Here again α_2 and α_3 are obtuse angle and therefore slopes of the tangents tan α_2 and tan α_3 are both negative, i.e, derivatives of the function at P_2 and P_3 are negative.

v) The function given by the curve (on the left) is a decreasing function.

vi) On the curve (on the right), take three points Q_1, Q_2 and Q_3 and the other straight wires, from tangents at each of these points making angles β_1, β_2, β_3, respectively with the positive direction of x-axis, as shown in the figure β_1, β_2, β_3 are all acute angles

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So the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

Observation :

$$\tan \alpha = m = \text{gradient}$$

$$y = f(x) = \frac{dy}{dx} = \tan \alpha$$

$\alpha_1 = 110° > 90°$, $\beta_1 = 70° < 90°$ $\tan \alpha_1$ = negative ; $\tan \beta_1$ = Positive

$\alpha_2 = 115° > 90°$, $\beta_2 = 75° < 90°$ $\tan \alpha_2$ = negative, $\tan \beta_2$ = Positive

$\alpha_3 = 120° > 90°$, $\beta_3 = 80° < 90°$ $\tan \alpha_3$ = negative, $\tan \beta_3$ = Positive

Application : This activity may be useful in explaining the concepts of decreasing and increasing function.

25.07.22

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Activity - 5

Objective: To understand the concepts of local maxima local minima and point of inflection.

Pre-Requisite Knowledge :

A. **First derevative method -**

i) If, for a perticular critical value $x = a$, the sign of $f'(x)$ changes from +ve to -ve as x increases through $x = a$, then $f(a)$ is a local maximum value.

ii) If for a particular critical value $x = a$, the sign of $f'(x)$ changes from -ve to +ve as x increases through $x = a$, then $f(a)$ is a local minimum value.

iii) If for a particular critical value $x = a$, the sign of $f'(x)$ does'nt changes as x increases through $x = a$, then $f(a)$ is neither a local maximum value nor a local minimum value.

B. **Second derevative method -**

For a particular critical value $x = a$,

i) $f''(a) < 0 \Rightarrow f(a)$ is a local maximum value.

ii) $f''(a) > 0 \Rightarrow f(a)$ is a local minimum value

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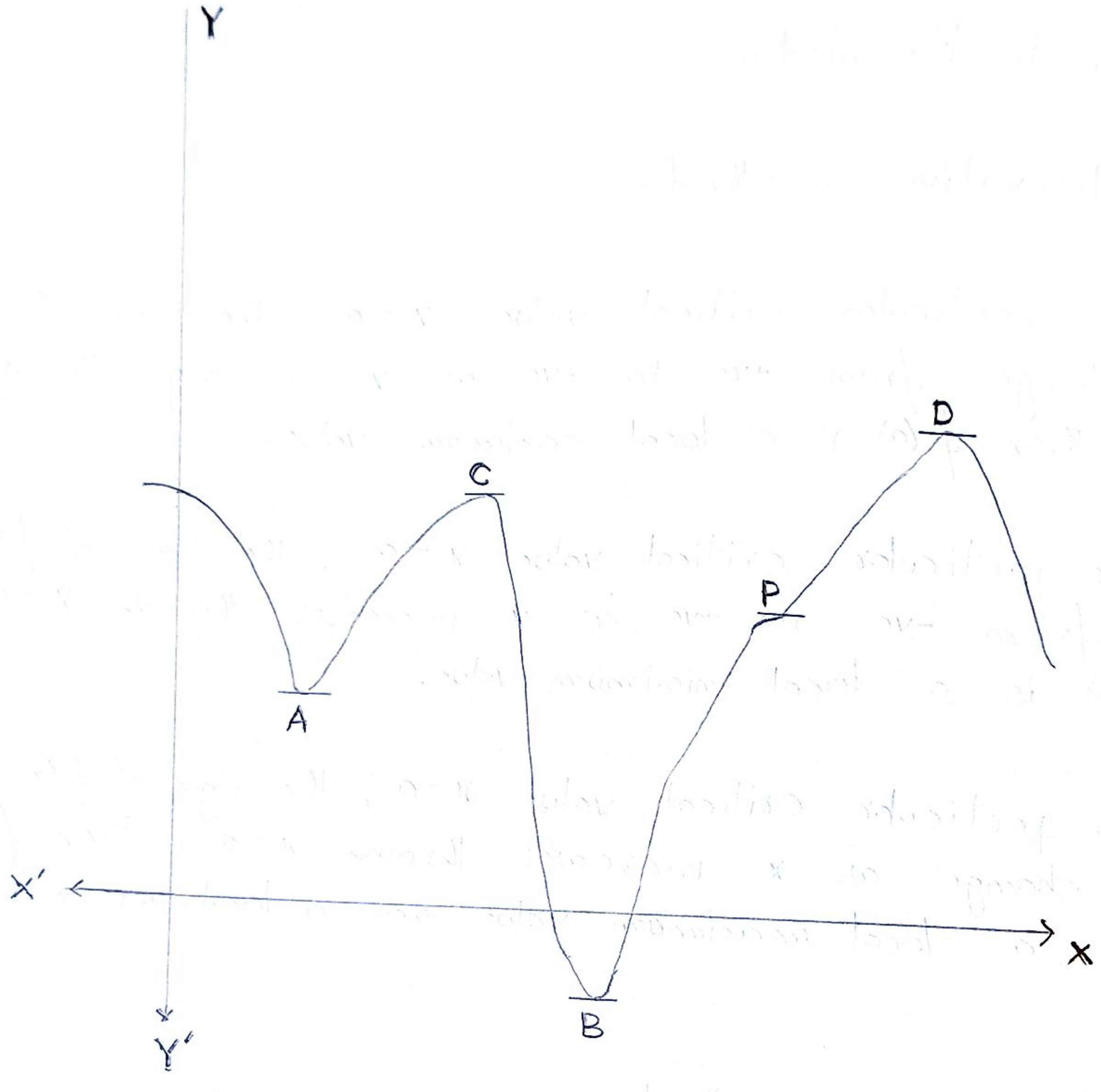
Y
C
D
P
A
X'
X
B
Y'

iii) $f''(a) = 0$ or ∞ $\Rightarrow$ The test fails and the first derivative method is used to study the nature of $f(a)$.

Materials Required: A piece of paper, fevicol, geometry box, white sheet, colour pens.

Procedure:

i) Take a white sheet of paper.
ii) Draw a horizontal and vertical line. Name them as x-axis and y-axis respectively
iii) Draw a curve on the white sheet as shown in figure
iv) Locate the points A, C, B, P and D on the curve as shown in the figure.
v) Draw a small horizontal line at each of these points A, C, B, P and D as shown in figure.

Observation:

i) Sign of the slope of the tangent i.e., first derevative at a point on the curve to the immediate neighbourhood of A and on the left of A is -ve.

ii) Sign of the slope of the tangent, i.e. first derivative at a point on the curve to the immediate neighbourhood of A and on the right of A is +ve.

iii) Sign of the first derivative at a point on the curve to immediate neighbourhood of B and left of B is -ve.

iv) Sign of the first derivative at a point on the curve to immediate neighbourhood of B and on the right of B is +ve.

v) Sign of the first derivative at a point on the curve to immidiate neighbourhood of C and on the left of C is +ve.

vi) Sign of the first derivative at a point on the curve to immidiate neighbourhood of C and on the right of C is −ve.

vii) Sign of the first derivative at a point on the curve to immidiate neighbourhood of D and on the left of D is +ve.

viii) Sign of the first derivative at a point on the curve to immidiate neighbourhood of D and on the right of D is −ve.

ix) Sign of the first derivative at a point immediate neighbourhood of P and on the left of P is +ve and immediate neighbourhood of P and on the right of P is +ve.

x) A and B are points of local minimum.

xi) C and D are points of local maximum.

xii) P is a point of inflection.

Result :

i) In the figure, the small horizontal lines at the point A, B, C and D represent tangents to the curve and are parallel to the axis.

ii) The slope of tangents at these points are zero, i.e., the value of the first derivative at these points is zero.

iii) The tangent at P intersects the curve.

iv) Sign of the first derivative at point A and B changes from negative to positive. So, they are the points of Local minima.

v) Sign of the first derivative at point C and D changes from positive to negative. So, they are the points of local maxima.

vi) Sign of first derivative at point P does'nt change. So, it is a point of inflection.

Practical Importance :

i) This activity may help in explaining the concept of point of Local maxima, minima and inflection.

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11) The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

02.09.22

Teacher's Signature

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Activity - 6

Objective : To verify geometrically that -
$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Pre-Requisite Knowledge :

i) Area of triangle with adjecent sides $\vec{a}$ and $\vec{b}$ = $\frac{1}{2}|\vec{a} \times \vec{b}|$

ii) Area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ = $|\vec{a} \times \vec{b}|$
where $|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Materials Required : Geometry box, cardboard, white paper, cotter, sketch pen, cellotape.

Procedure :

i) Take a cardboard and paste a white paper on it.

ii) Draw a line segment OA = 6 cm (say) and let it represent $\vec{a}$.

iii) Draw another line segment OB = 4 cm (say) at an angle 60° with OA. Let $\vec{OB} = \vec{b}$

iv) Draw BC = 6 cm (say) at an angle 30° with $\vec{OA}$. Let $\vec{BC} = \vec{c}$.

v) Draw perpendicular BM, CL and BN.

vi) Complete the parallelogram OAPC, OAQB and BQPC.

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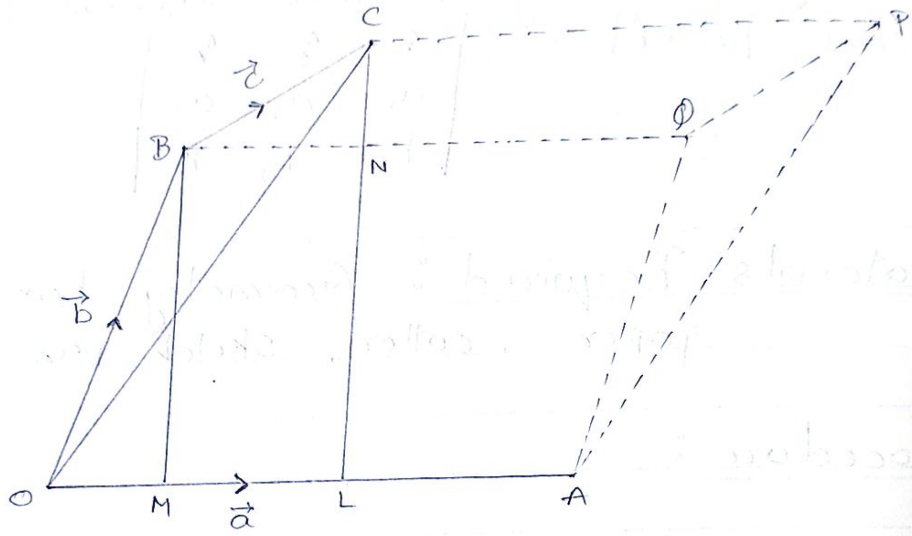

C
P
$\vec{c}$
B
Q
N
$\vec{b}$
O
M
$\vec{a}$
L
A

Demonstration:

i) $\vec{OC} = \vec{OB} + \vec{BC} = \vec{b} + \vec{c}$ and let $\angle COA = \alpha$

ii) $|\vec{a} \times (\vec{b} + \vec{c})| = |\vec{a}|\,|\vec{b} + \vec{c}| \sin\alpha$ = Area of parallelogram (OAPC)

iii) $|\vec{a} \times \vec{b}|$ = Area of parallelogram OAQB

iv) $|\vec{a} \times \vec{c}|$ = Area of parallelogram BQPC

v) Area of parallelogram OAPC $= (OA)(CL)$
$= (OA)(LN + NC)$
$= (OA)(BM + NC)$
$= (OA)(BM) + (OA)(NC)$

Where $(OA)(BM) + (OA)(NC)$
= Area of parallelogram OAQB + Area of parallelogram BQPC.
$= |\vec{a} \times \vec{b}| + |\vec{a} \times \vec{c}|$

$\therefore |\vec{a} \times (\vec{b} + \vec{c})| = |\vec{a} \times \vec{c}| + |\vec{a} \times \vec{b}|$

Direction of each of these vectors $\vec{a} \times (\vec{b} + \vec{c})$, $\vec{a} \times \vec{b}$ and $\vec{a} \times \vec{c}$ are perpendicular to the same plane.
Hence $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Observation:

$|\vec{a}| = |\vec{OA}| = OA = 6$ cm

$|\vec{b}+\vec{c}| = |\vec{OC}| = OC = 7\ cm$

$CL = 5\ cm$

$|\vec{a} \times (\vec{b}+\vec{c})|$ = Area of parallelogram OAPC
$= (OA)\ (CL) = 6 \times 5 = 30$ sq. cm.

$|\vec{a} \times \vec{b}|$ = Area of parallelogram OAQB
$= (OA)\ (BM) = 6 \times 3.5 = 21$ sq. cm.

$|\vec{a} \times \vec{c}|$ = Area of parallelogram BQPC
$= (QA)\ (CN) = 6 \times 1.5 = 9$ sq. cm.

From (i), (ii) and (iii),
Area of parallelogram OAPC
= Area of parallelogram OAQB + Area of parallelogram BQPC

$\therefore |\vec{a} \times (\vec{b}+\vec{c})| = |\vec{a} \times \vec{b}| + |\vec{a} \times \vec{c}|$

$\vec{a} \times \vec{b}$, $\vec{a} \times \vec{c}$ and $\vec{a} \times (\vec{b}+\vec{c})$ are all in the direction of normal to the plane of paper.

$\therefore \vec{a} \times (\vec{b}+\vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Result : We have verified : $\vec{a} \times (\vec{b}+\vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

18.08.22

Date

Expt. No.

Activity - 7

Objective : To verify that angle in a semi-circle is a right angle, using vector method.

Pre-Requisite Knowledge :

i) Addition of vector
- Triangle law of addition
- Parallelogram law of addition.

ii) Difference of vectors

iii) Multiplication of vectors.

Materials required : White paper, geometry box, Colour pens, adhesive, etc.

Procedure :

i) Take a white sheet of paper

ii) Draw a circle on the paper, with centre O and radius of 5 cm.

iii) Take the points A, B, P, Q and R on the circle.

iv) Join OP, OA, OB, AP, AQ, BQ, OQ, OR, AR, BR, BP using a ruler.

v) Put arrows on OA, OB, OP, AP, BP, OQ, AQ, BQ, OR, AR, and BR to show that as vector.

vi) Paste it on a practical copy.

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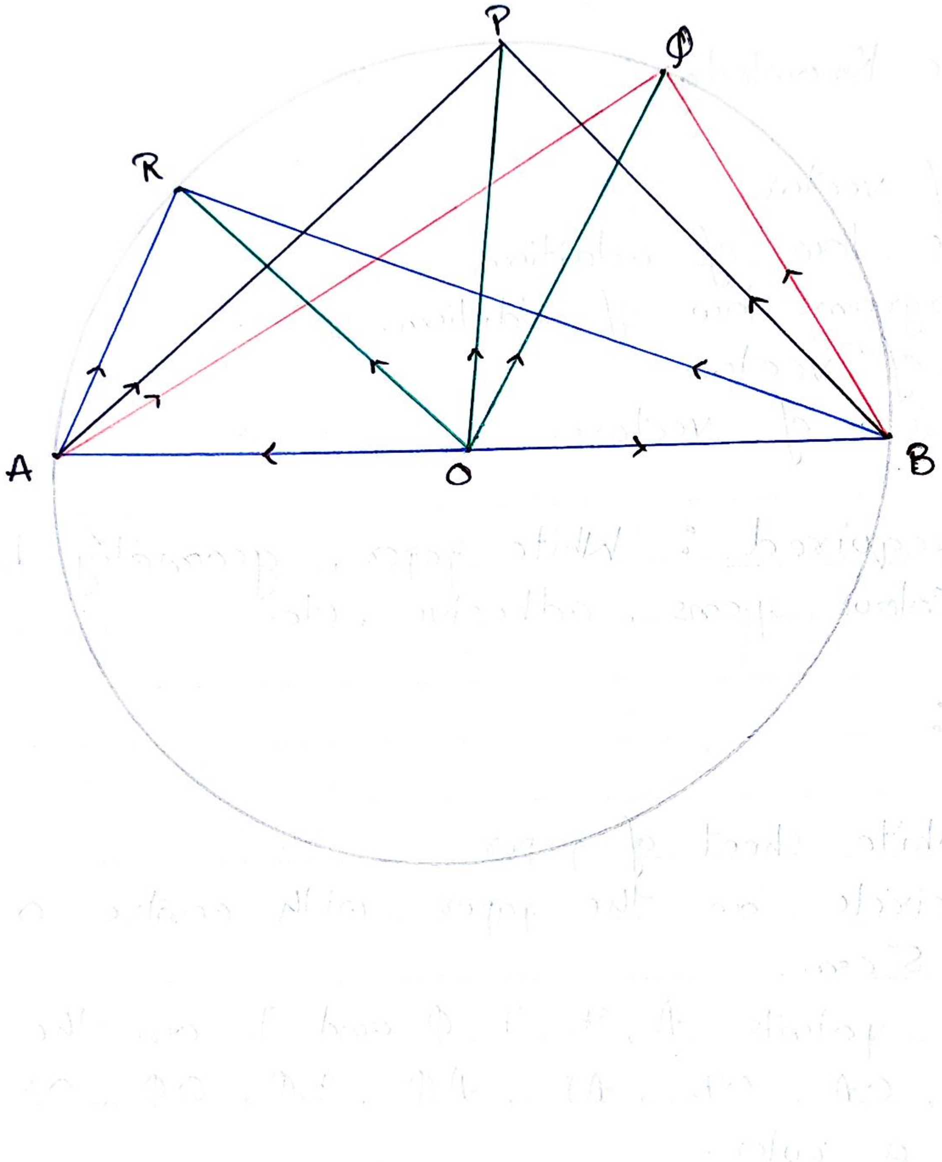
P
Q
R
A
O
B

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Demonstration :

i) The point A, B, P, Q and R all lie on the same semicircle.
ii) Measure the angle between the vectors $\vec{AP}$ and $\vec{BP}$ using a protactor i.e., $\angle APB = 90^\circ$
iii) Similarly, measure the angle between the vectors $\vec{AQ}$ and $\vec{BQ}$, i.e., $\angle AQB = 90^\circ$
iv) Also, the angle between the vectors $\vec{AQ}$ and $\vec{BR}$, is $\angle ARB = 90^\circ$
v) All angles formed between two vectors in a semi-circle is a right angle.

Observation :

Let $|\vec{OA}| = |\vec{OB}| = a$ (radius of the circle)
and $|\vec{OP}| = p$, $|\vec{OQ}| = q$, $|\vec{OR}| = r$

By actual measurement,
$|\vec{OA}| = |\vec{OB}| = |\vec{OP}| = |\vec{OQ}| = |\vec{OR}|$
$\Rightarrow a = p = q = r = 5$ cm

$|\vec{AP}| = 7.1$ cm, $|\vec{BP}| = 7.1$ cm, $|\vec{AB}| = 10$ cm, $|\vec{AQ}| = 8$ cm,
$|\vec{BQ}| = 6$ cm, $|\vec{AR}| = 4.4$ cm, $|\vec{BR}| = 9$ cm.

$|\vec{AP}|^2 + |\vec{BP}|^2 = (7.1)^2 + (7.1)^2 = 100 = |\vec{AB}|^2$
$|\vec{AQ}|^2 + |\vec{BQ}|^2 = (8)^2 + (6)^2 = 100 = |\vec{AB}|^2$
$|\vec{AR}|^2 + |\vec{BR}|^2 = (4.4)^2 + (9)^2 = 100 = |\vec{AB}|^2$

So $\angle APB = 90^\circ$ and $\vec{AP} \cdot \vec{BP} = 0$, $\angle AQB = 90^\circ$ and

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$\overrightarrow{AQ} \cdot \overrightarrow{BQ} = 0$, and $\angle ARB = 90°$ and $\overrightarrow{AR} \cdot \overrightarrow{BR} = 0$.

Thus we have angle in a semicircle is a right angle.

Alternatively :

Let $\overrightarrow{OA} = \overrightarrow{OB} = \vec{a} = \overrightarrow{OP} = \vec{p}$

$\overrightarrow{OA} = -\vec{a}$; $\overrightarrow{OB} = \vec{a}$, $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$. $\overrightarrow{BP} = \vec{p} - \vec{a}$

$\overrightarrow{AP} \cdot \overrightarrow{BP} = (\vec{p} + \vec{a}) \cdot (\vec{p} - \vec{a}) = |\vec{p}|^2 + |\vec{a}|^2 = 0$

Therefore the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ} \cdot \overrightarrow{BQ} = 0$, so, $\angle AQB = 90°$

$\overrightarrow{AR} \cdot \overrightarrow{BR} = 0$, so, $\angle ARB = 90°$

Result : Using vector method, we have verified that angle in a semicircle is a right angle

Practical Importance :

This activity can be used to explain the concepts of -

i) Opposite vectors
ii) vectors of equal magnitude
iii) Perpendicular vectors
iv) dot product of two vectors.

21.10.22

ACTIVITY - 8

Objective: Give the solution to the following activities of Cartesian and vector form of presentation the equation of lines.

i) The cartesian equation of a line are $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-3}{5}$ Find vector equation for the line.

ii) The cartesian equation of a line are $6x-2 = 3y+1 = 2z-2$ Find the direction ratios, and also find a vector equation for the line.

iii) A line passes through the point with position vector, $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is in the direction ratios of $3\hat{i} + 4\hat{j} - 5\hat{k}$ Find the equation of the line in vector and cartesian form.

iv) Find the vector equation of the line through $A(3,4,-7)$ and $B(1,-1,6)$. Find also its cartesian equation.

v) The points $A(4,5,10)$, $B(2,3,4)$ and $C(1,2,-1)$ are three vectors of a parallelogram. find vector and cartesian equation of AB and BC, and find coordinates of D.

Pre-Requisite Knowledge:

i) Equation of Cartesion form: If a line passes through the points (x_1, y_1, z_1) and has direction ratios b_1, b_2, b_3 then its equation in Cartesian form are -
$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$$

ii) Equations in Vectorial form: If a line passes through the point (x_1, y_1, z_1) and has direction ratios

b_1, b_2, b_3 then its equation in vectorial form is-
$$\vec{r} = \vec{a} + \lambda\vec{b}$$
or, $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

Materials required: Practical workbook, pencil, ruler, eraser.

Procedure: The solution to the problems is as under -

i) The equation of a line is $\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z-3}{5}$, it also passes through $(3, -1, 3)$ and is parallel to the line whose direction ratios are $2, -2$ and 5.

$\therefore$ The vector equation is $\vec{r} = 3\hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + 5\hat{k})$ where λ is parameter.

ii) The cartesian equation of a line is $6x-2 = 3y+1 = 2z-2$. To put it in symmetrical form, we must make the coefficients of x, y, z as 1. Thus we obtain,
$\frac{x - 1/3}{1} = \frac{y + 1/3}{2} = \frac{z-1}{3}$. This shows that the given line passes through $(1/3, -1/3, 1)$ and is parallel to the line whose direction ratios are $1, 2, 3$.
$\therefore$ Its vector equation is $\vec{r} = (1/3)\hat{i} - (1/3)\hat{j} + \hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ where λ is parameter.

iii) Since the line passes through $2\hat{i} - 3\hat{j} + 4\hat{k}$ and has the direction ratios of $3\hat{i} + 4\hat{j} - 5\hat{k}$.

$\therefore$ Its vector equation is given by, $\vec{r} = (\vec{a} + \lambda\vec{b})$, or,
$\vec{r} = 2\hat{i} - 5\hat{j} + 4\hat{k} + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$ where λ is parameter.
The cartesian equivalent of the above equation is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

iv) Since the line through $A(3,4,-7)$ and $B(1,-1,6)$. Its position vector is given by $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\hat{i} - \hat{j} + 6\hat{k}$ and the equation is -

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \text{ where } \lambda \text{ is a parameter}$$

$\therefore \vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda[(\hat{i} - \hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} - 7\hat{k})]$

$\therefore \vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda[2\hat{i} + 5\hat{j} - 13\hat{k}]$ where λ is parameter
The cartesian equation equivalent of (i) is -

$$\frac{x-3}{2} = \frac{y-4}{5} = \frac{z-(-7)}{-13}$$

$$\Rightarrow \frac{x-3}{2} = \frac{y-4}{5} = \frac{z+7}{-13}$$

v) As the line passes through $A(4,5,10)$ and $B(2,3,4)$ its vector equation is - $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\therefore \vec{r} = 4\hat{i} + 5\hat{j} + 10\hat{k} + \lambda[(2\hat{i} + 3\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + 10\hat{k})]$

$\Rightarrow \vec{r} = 4\hat{i} + 5\hat{j} + 10\hat{k} + \lambda(-2\hat{i} - 2\hat{j} - 6\hat{k})$

$\therefore \vec{r} = 4\hat{i} + 5\hat{j} + 10\hat{k} - 2\lambda(\hat{i} + \hat{j} + 3\hat{k})$

$\Rightarrow \vec{r} = 4\hat{i} + 5\hat{j} + 10\hat{k} + t(\hat{i} + \hat{j} + 3\hat{k})$

where t is replaced by -2λ

The cartesian equation equivalent of (1) is $\frac{x-4}{1} = \frac{y-3}{1} = \frac{z-4}{5}$

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Next as BC passes through $B(2,3,4)$ and $C(1,2,-1)$ its vector equation is -

$\therefore \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu[(\hat{i}+2\hat{j}-\hat{k}) - (2\hat{i}+3\hat{j}+4\hat{k})]$

$\Rightarrow \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu(-\hat{i}-\hat{j}-5\hat{k})$

$\Rightarrow \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k} + s(\hat{i}+\hat{j}+5\hat{k})$

where $s = -\mu$

The cartesian equation equivalent of (II) is -

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{5}$$

Let the coordinates of D be (x, y, z). Since ABCD is a parallelogram, diagonals AC and BD bisects and other as shown in the figure, Therefore AC and BD must have the same midpoint. Midpoint of AC is -
$M\left(\frac{4+1}{2}, \frac{5+2}{2}, \frac{10-1}{2}\right)$; i.e. ; $M\left(\frac{5}{2}, \frac{7}{2}, \frac{9}{2}\right)$ and midpoint of BD is $N\left(\frac{x+2}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right)$. So M and N are the same point, we have, $\left(\frac{x+2}{2} = \frac{5}{2}, \frac{y+3}{2} = \frac{7}{2}, \frac{z+4}{2} = \frac{9}{2}\right)$. Π

Thus the coordinates of D are $(3,4,5)$.

Results: The result of the desired activities are -

1) The cartesian equation of line $\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z-3}{5}$, has vector equation as $\vec{r} = \hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + 5\hat{k})$

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where λ is parameter.

ii) The cartesian equation of a line $6x-2=3y+1=2z-2$, has vector equation as $\vec{r}=\frac{1}{3}\hat{i}-\frac{1}{3}\hat{j}+\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$

where λ is parameter, whose direction ratio are 1, 2, 3.

iii) Lines with position vector, $2\hat{i}-3\hat{j}+4\hat{k}$ and $3\hat{i}+4\hat{j}-5\hat{k}$ here cartesian equivalent of above equation is -

$$\frac{x-2}{3}=\frac{y+3}{4}=\frac{z-4}{-5}$$

iv) The cartesian equation of line through $A(3,4,-7)$ and $B(1,-1,6)$ is $\frac{x-3}{2}=\frac{y-4}{5}=\frac{z+7}{-13}$

v) Points $A(4,5,10)$ and $B(2,3,4)$ and $C(1,2,-1)$ of a parallelogram will have the coordinates of $D(3,4,5)$

Practical Importance:

This exercise demonstartes, the good exposition of cartesian and vector form of equations of the line. The students learn to express a line in any of the forms desired for solving a problem. The solutions presented here-in, forms a procedural tool and this experiment will make the concept very clean.

16.11.22

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ACTIVITY - 9

Objective : A furniture dealer deals in only two items namely, tables and chairs. He has ₹5000 to invest and a space to store, at the most 60 pieces. A table costs him ₹250 and a chair ₹50. He call sell a table, at a profit of ₹50 and a chair at a profit of ₹15. Assuming that he can sell all the items that he buys, how should he invest his money in order that he may maximise the profit?

Pre-Requisite Knowledge :

i) A relationship of the form $ax + by + c < 0$ or $ax + by + c \leq 0$ or $ax + by + c > 0$ or $ax + by + c \geq 0$, where, a, b, c are arbitary constants is called a linear inequation or inequality.

ii) Feasible Region : The region which is common to all constraints of a linear programming problem (LPP) is called feasible region of the given LPP.

iii) Feasible Solution : Every point in the feasible region of a linear programming problem (LPP) is called a feasible solution of the given LPP.

Materials Required : Drawing sheet or a graph paper, geometry box, measuring scale, practical workbook.

Procedure :

1) Let x be the number of tables, and y the number of chairs that he buys.

Clearly, $x \geq 0$ and $y \geq 0$. Also we have $x+y \leq 60$
Cost of a table = ₹250
Cost of a chair = ₹50
Since he can trust and invest at the most ₹5000

We have, $250x + 50y \leq 5000$
$5x + y \leq 100$

Profit on a table = ₹50
Profit on a chair = ₹15
Let, P be the profit function on the investment
$P = 50x + 15y$

Mathematically, the above said problem reduces to maximise, profit, $P = 50x + 15y$
subject to, $x \geq 0$, and $y \geq 0$, $x+y \leq 60$ \$ $5x+y \leq 100$

To draw the graph for $5x+y=100$, $x+y=60$, the values –

Equations				
$5x+y=100$	x	20	0	10
	y	0	100	50
$x+y=60$	x	0	60	10
	y	60	0	50

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The solution of System A, is required shaded feasible region OABC whose boundary points are O(0,0), A(20,0), B(10,50) and C(0,60)

Since the maximum or minimum occurs only at the boundary points and so let us calculate the value of the profit of the feasible region.

Sl. No.	Boundary pts. of feasible region	$P = 50x + 15y$
1.	O (0,0)	$P = 50\times0 + 15\times0 = 0$
2.	A (20,0)	$P = 50\times20 + 15\times0 = 1000$
3.	B (10,50)	$P = 50\times10 + 15\times50 = 1250$
4.	C (0,60)	$P = 50\times0 + 15\times60 = 900$

Boundary points B, gives the maximum profit = ₹ 1250, therefore, the dealer should produce 10 tables and 50 chairs in order to get maximum profit.

Result: Shaded region given in the figure represents the solution for the set of given linear inequations, and it is concluded that the maximum profit will be earned, when 10 tables and 50 chairs are sold, the profit will be ₹ 1250.

Precautions: i) Line work should be neat and tidy.
ii) Avoid erasing as far as possible.

Practical Importance: The activity discussed here in is a practice problem and the students with the solutions, be benifited herein.

24.11.22

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Activity - 10

Objective : To explain the computation of conditional probability of a given event A, when event B has already occured, through a example of throwing a pair of dice.

Materials Required : A peice of cardboard, white sheet, pencil, scale, a pair of dice, glue, etc.

Pre - Requisite Knowledge :

1) Concept of sample space
2) Concept of conditional probability

Method of Construction :

1) Fix a white sheet on a piece of cardboard of a suitable size.
2) Make a square and divide it into 36 unit squares of 1.5 cm
3) Write the pairs of number.

Demonstration :

1) All possible outcomes of the given experiment are shown in the figure. Hence, it gives the sample space of the experiment.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6.

2) Suppose we have to find the conditional probability of an event A if an event B has already occurred where A is the event "a number 4 appears on both the dice" and B is the event "4 has appeared atleast one of the dice" i.e. we have to find $P(A/B)$

3) From figure, number of outcomes favourable to A = 1
Number of outcomes favourable to B = 11
Number of outcomes favourable to $A \cap B$ = 1

4) (i) $P(B) = 11/36$
(ii) $P(A \cap B) = 1/36$
(iii) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{11}$

Observation:

1) Outcomes favourable to A : 1
2) Outcomes favourable to B : 11
3) Outcomes favourable to $A \cap B$: 1
4) $P(A \cap B) = 1/36$
5) $P(A/B) = 1/11$

Result: We have explained how to compute conditional probability of an event.

Practical Importance: This activity is very helpful to explain the concept of conditional probability, which is further used in Bayes' Theorem.

Teacher's Signature 5.12.22